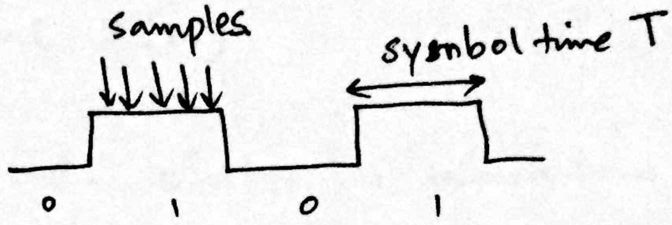
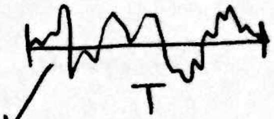
OFDM

→ data bits 0, 1, 0, 1



- → many samples for same bit ⇒ wasteful.
- → OFDM wants to make every sample useful.

→ data bits ⟶ complex freq. ⟶ $\boxed{\text{IFFT or IDFT}}$ ⟶ time series



contains many frequencies encoding information

→ So what is the lowest & highest freq. we can encode?

Lowest $= \frac{1}{T}$ say T is defined in samples.

Highest $= \frac{1}{2} f_s$ Nyquist.

→ From the perspective of IDFT.

$$\underbrace{X(m)}_{\text{freq. samples}} \longrightarrow \boxed{\text{IDFT}} \longrightarrow \underbrace{x(n)}_{\text{time samples}}$$

IDFT ↓ N point IDFT

$\boxed{N = \text{length of the symbol}}$

$$\underbrace{x(n)}_{\substack{\text{complex} \\ \text{time series} \\ \text{(N samples)}}} = \frac{1}{N} \sum_{m=0}^{N-1} \underbrace{X(m)}_{\substack{\text{complex} \\ \text{freq.} \\ \text{(N samples)}}} e^{j 2\pi/N \cdot m n}$$

→ Let's look at different $X(m)$

- → $X(0)$ is DC.
- → Setting $X(0)$ to a value shifts the whole time series.

i.e.

$$x_n = \frac{1}{N}\sum_{m=0}^{N-1} X(m)\, e^{j2\pi/N \cdot m \cdot n}$$

for $m=0$, $X(m=0)$ will get added like an offset to all values of x_n.

i.e.,

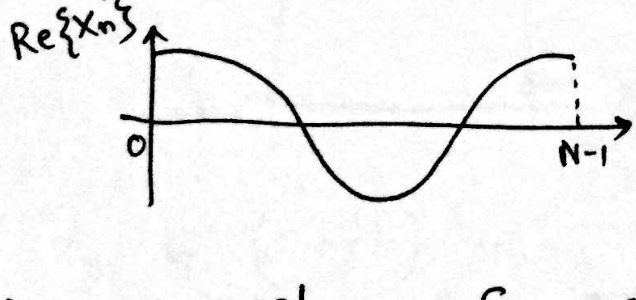
$$X(n) = X(m=0) + \cancel{X(m=1) + X(m=2) \ldots}$$
$$X(m=1)\cdot e^{j2\pi/N \cdot n} +$$
$$X(m=2)\cdot e^{j2\pi/N \, 2n} +$$
$$\ldots$$

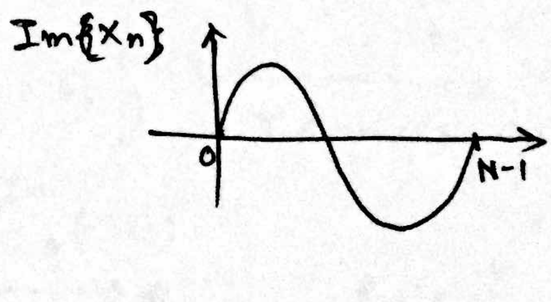
→ This is a constant bias.

→ Now look at $X(m=1)$

- → This is lowest non-trivial freq.
- → If all other m's are 0's, then $X(n)$ will be $X(m=1)\cdot e^{j2\pi/N \cdot 1 \cdot n}$ which is

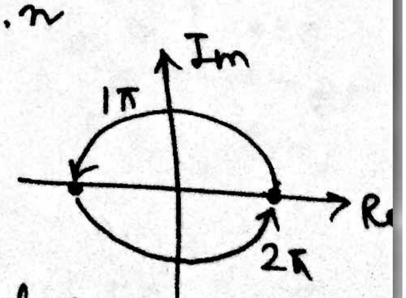
$$X(n) = X(m=1)\left(\cos\theta + j\sin\theta\right). \quad \text{where } \theta = \frac{2\pi}{N}\cdot n.$$



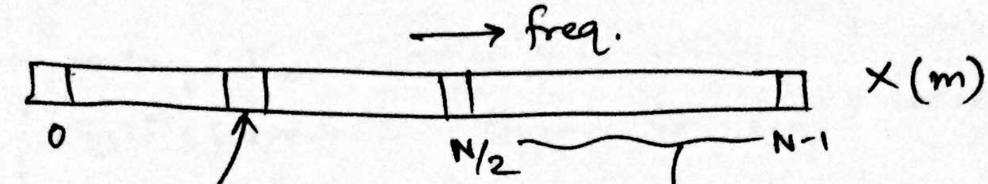


→ $X\left(\frac{N}{2}\right)$ → we get max freq. since

$$X(n) = \sum X(m)\, e^{j2\pi/N \cdot \left(m=\frac{N}{2}\right)\cdot n}$$
$$= \sum X_m \cdot e^{jn\pi} \equiv$$



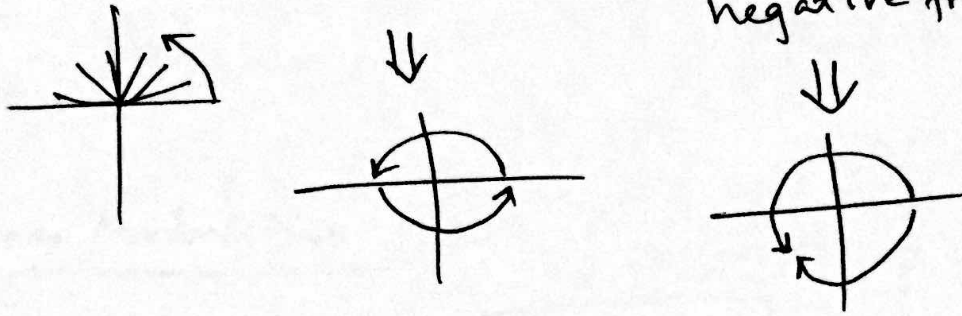
$n\pi$ means the signal switches back and forth.



$F = \frac{K}{N} f_s$ ⇓

Highest freq = Nyquist ⇓

mirror freq. or negative freq. ⇓

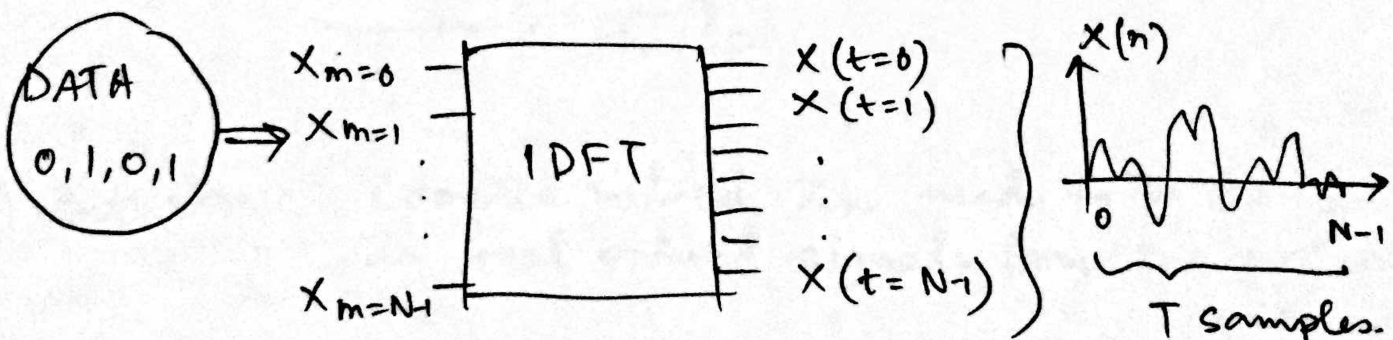


The counter clock wise can be equivalent to clockwise.

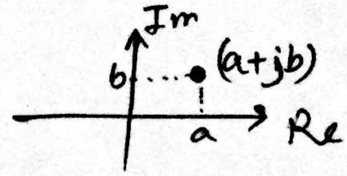
positive freq.

Negative freq.

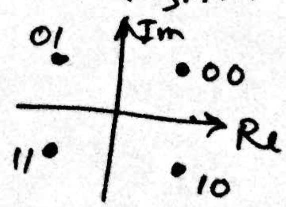
OFDM Symbol Generation



Now X_m is $(a+jb)$ =



4QAM



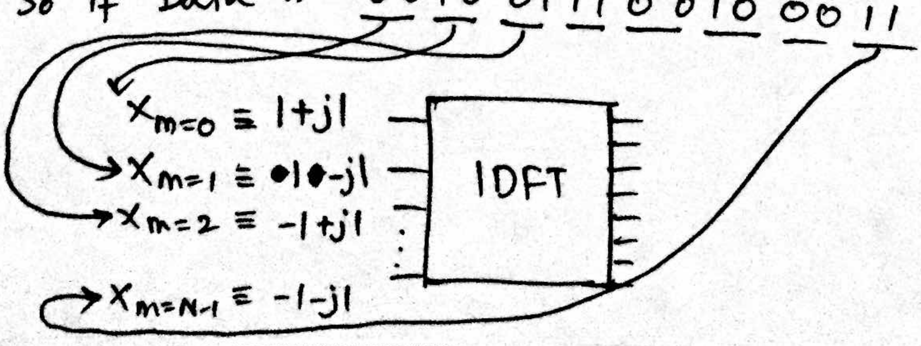
So if Data is 00 10 01 11 00 10 10 00 11

$X_{m=0} \equiv 1+j1$

$X_{m=1} \equiv 1-j1$

$X_{m=2} \equiv -1+j1$

$X_{m=N-1} \equiv -1-j1$



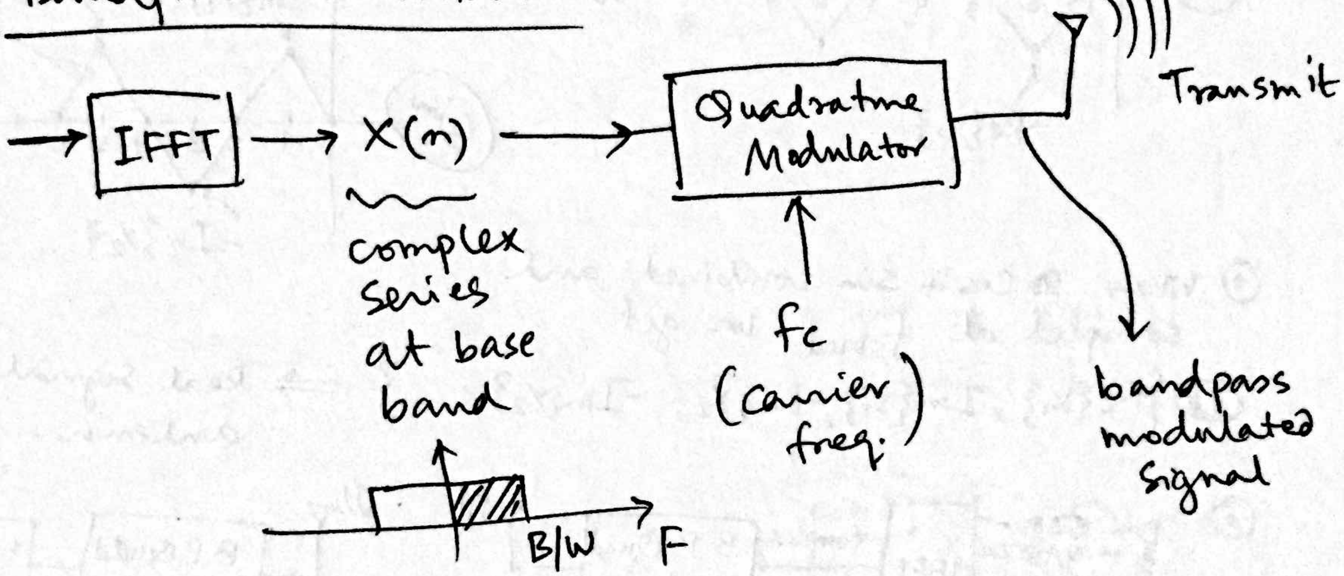
or 16 QAM

or 256 QAM. . .

→ How many X_m should I choose? i.e., what is N?

- ↳ N should be power of 2 so we can do Fast DFT (IFFT)
- ↳ Function of the bandwidth (??)

→ Bandpass Modulation

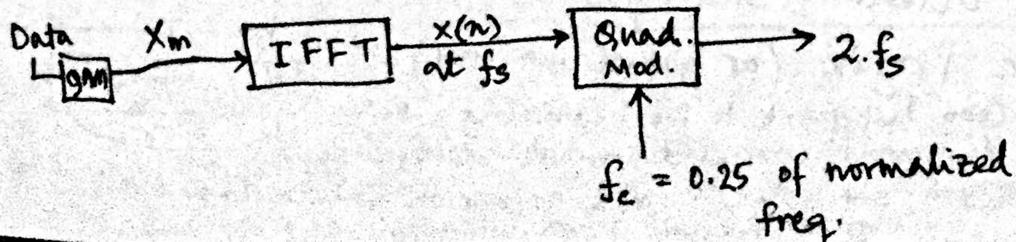


→ But note: Complex valued X_n needs to be transmitted as real valued signals from the antenna.

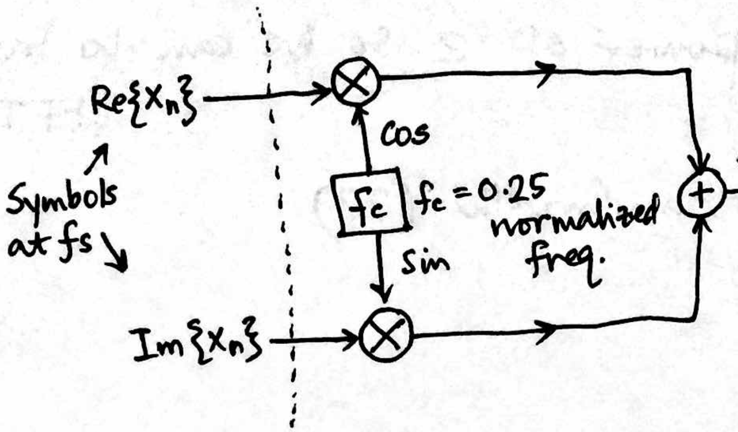
→ Quad Mod: Use f_m'' as the lowest possible freq. (ie., The modulating signal is greater than msg sig.)

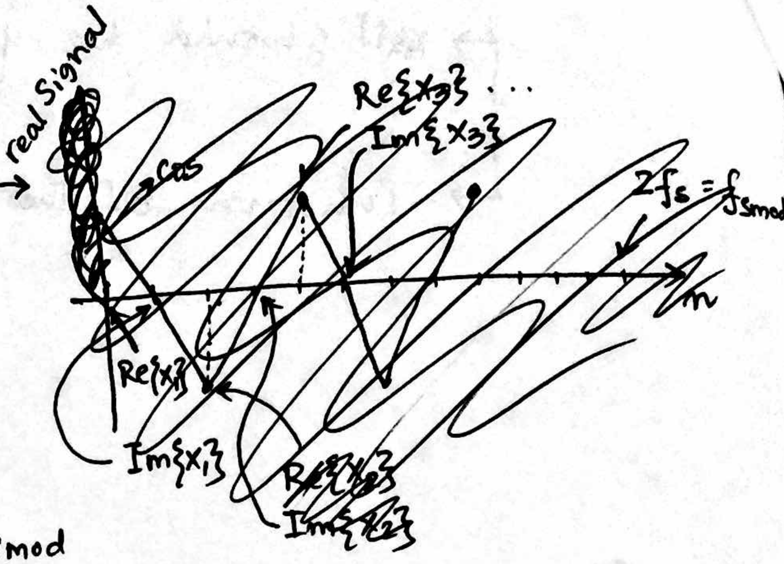
<u>Idea</u>: Take $x(n) = \{x_1, x_2, x_3 \dots\}$ and interleave the Re and Im values. i.e., $Re\{x_1\}, Im\{x_1\}, -Re\{x_2\}, -Im\{x_2\}, Re\{x_3\} \dots$

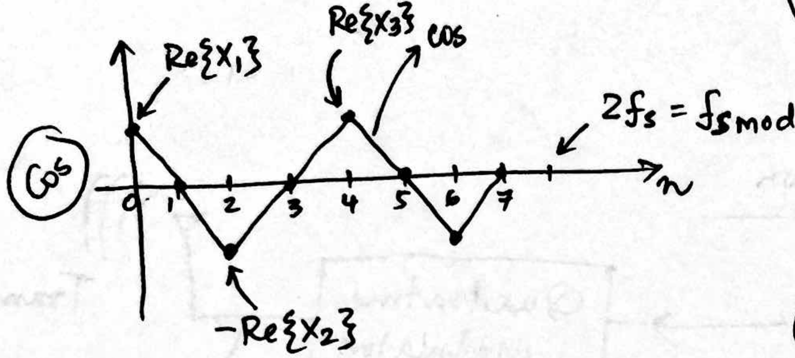
So, f_m'', also denoted as $f_{smod} = 2 \cdot f_s$ where f_s is highest freq. of $X(n)$. Use f_{smod} to modulate f_c.

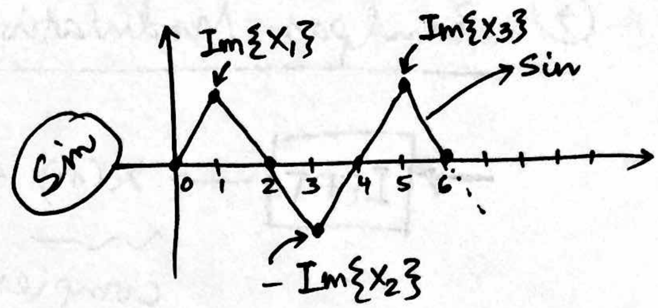


Bandpass Mod. cont.





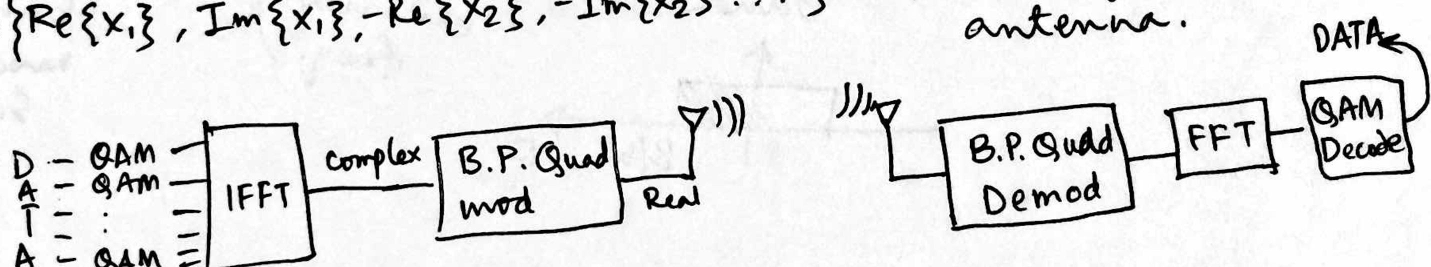




→ When Cos + Sin combined and sampled at f_{smod}, we get

$\{Re\{X_1\}, Im\{X_1\}, -Re\{X_2\}, -Im\{X_2\} \ldots\}$ ⟹ Real signal sent over antenna.

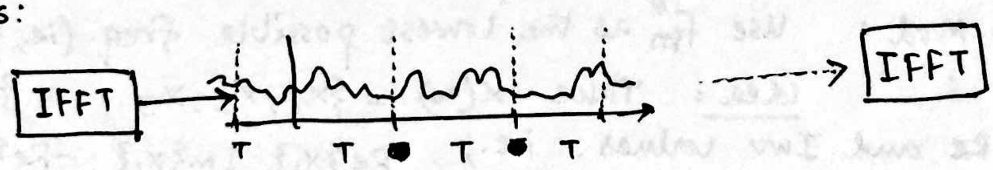
→



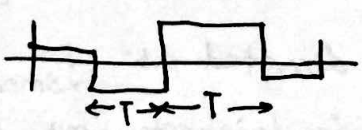
→ However: $\boxed{\text{FFT}}$ needs to know the start of the symbol accurately.

↳ the perfect sample.

That is:



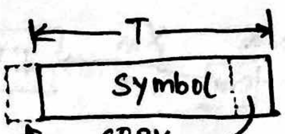
→ In BPSK, its easy



→ Symbol Start Detection Strategies

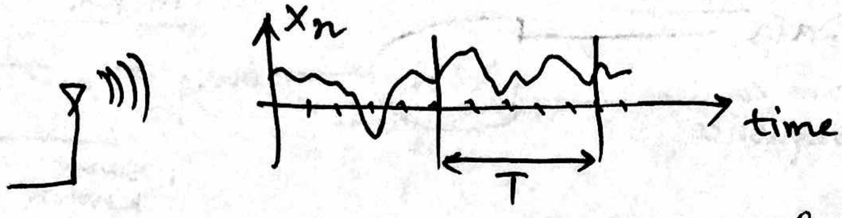
① Cyclic Prefix (or guard interval)

↳ copy last part to the beginning & then do cross correlation with delayed sig.



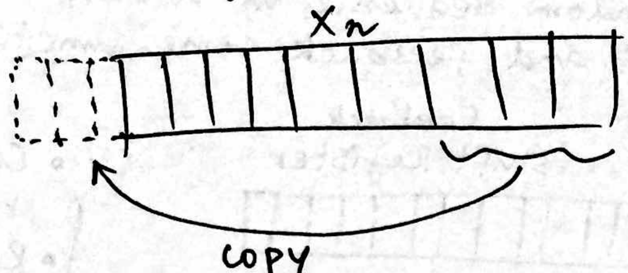
② Pilot tones : Set some $X(m)$'s to known values. (e.g., $X(22) = 2$)

Then at Rx, we know what to expect for X_{22}.

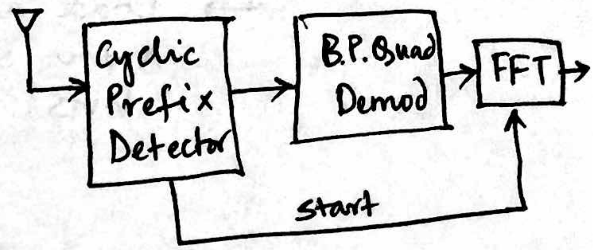


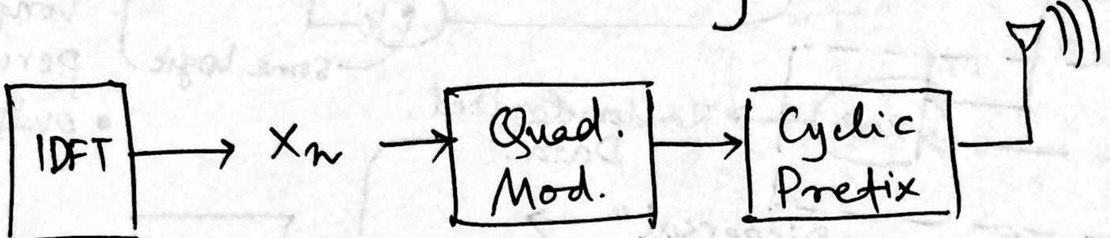
How do you know where symbols start ?

- Idea: Cyclic Prefix

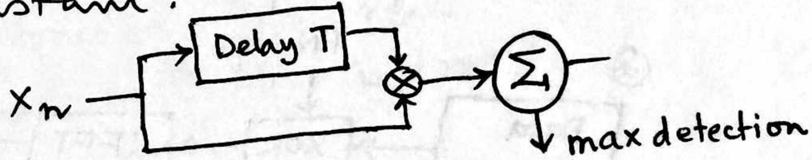


At Receiver:

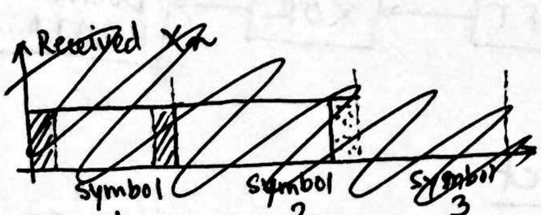


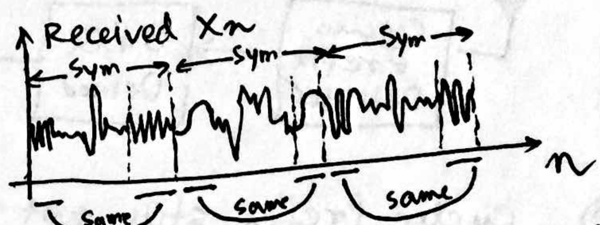


- Start Detection
  - Cross correlation to obtain start of symbol.
    ↳ fails when signal is constant because data is constant.
  - Cross correlation : (window of cyclic prefix)



- But if received symbol is constant, the cross-correlation also constant → no spikes → failed to detect start.





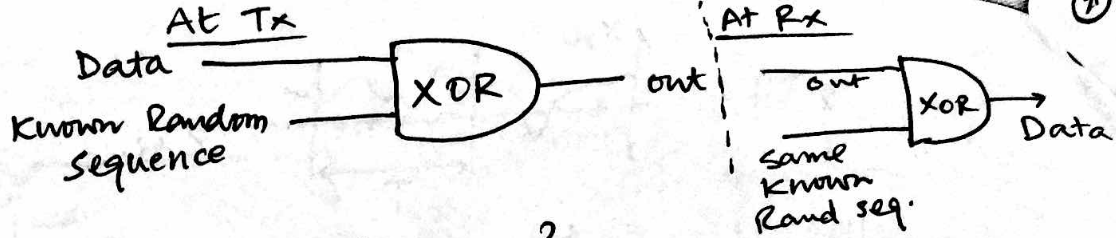
Ideal signal should be close to random sig so that cross correlation stays at ZERO except when prefix identical.

Example of worst case: Data = {1,1,1,1....} $\xrightarrow{\text{IDFT}}$ $\underbrace{\{1,0,0,0,0\ldots\}}_{\text{delta pulse}}$

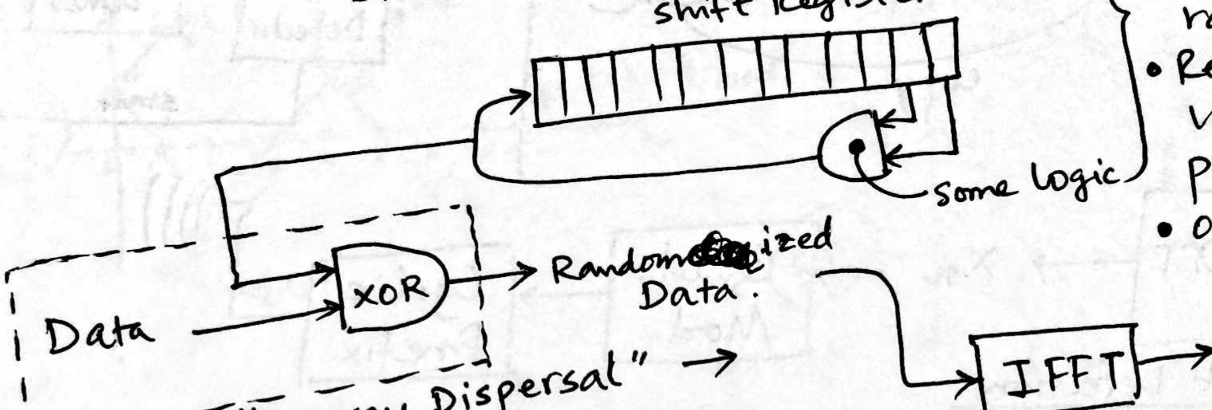
Example of best case: Data = {1,0,0,1,0,1,1} → Random sequence

- But how to make Data sequence look random ??

- **Idea:**



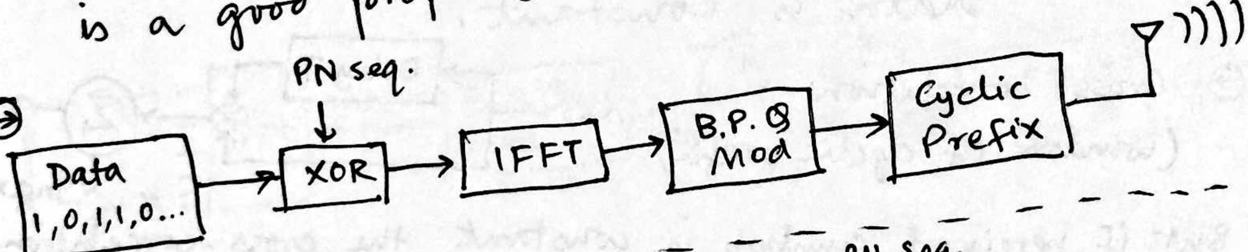
- How to create known random seq.?
  - Simple Idea : Shift Register
    - Load a random sequence in a shift register, then shift and feedback some function of output bits.

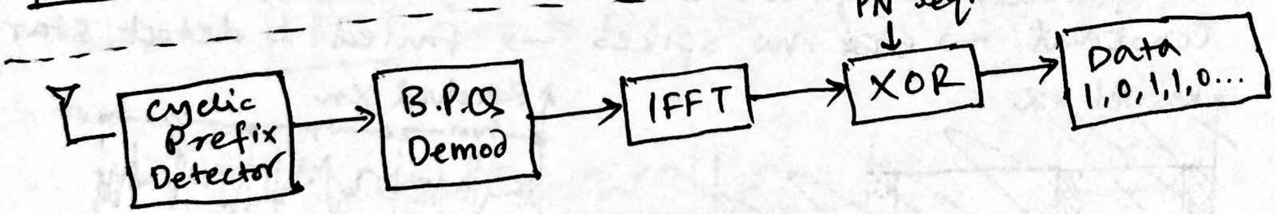


- Creates pseudo random seq.
- Repeats after very long seq (i.e., period very long).
- output random.

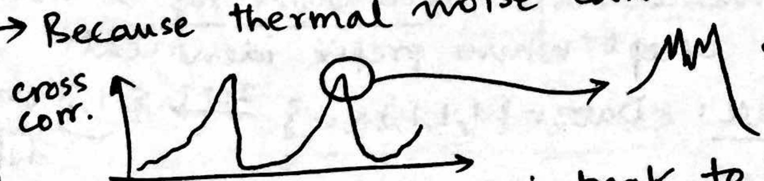
called "Energy Dispersal"

- Energy Dispersal spreads energy across the spectrum, which is a good property.





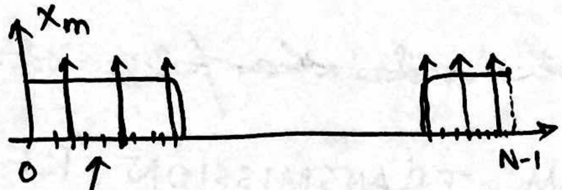
- Cyclic Prefix still not perfect
  - Because thermal noise can make the spike "smudged".



  - However, we need this peak to be exactly the correct **SAMPLE**

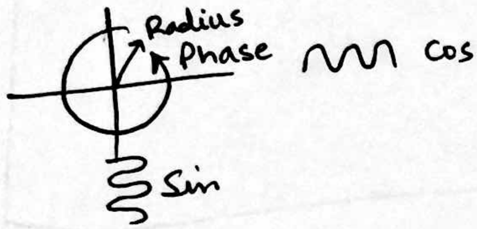
- Solution
  - ① coarse grained start detection from ~~[illegible]~~ cyclic prefix
  - ② Use pilot tones to fine tune to one sample

→ Pilot tones :



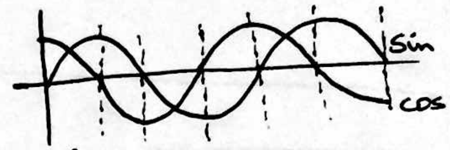
Each freq. contains QAM coded data.
Except some contains known amp. & phase called pilots

→ Receiver Side: Precise start detection possible becamse :
a pilot tone is a complex cos/sin wave,



If you start at wrong sample, the phase will be badly messed up.

i.e.,



You start wrong with screw up phase.

⇓ FFT

X(m)

→ So strategy
→ Make pilot tone $X(k) = 1 + j0$ ⇒ Real.
→ Then, shift the start of the FFT until the pilot tones are all real
→ Keep shifting until this is true.

⑦ FULL OFDM TRANSMISSION / RECEPTION