OFDM

3. data bits 0, 1, 0, 1
   many samples for same bit ⇒ wasteful.
   OFDM wants to make every sample useful.

3. data bits \( \rightarrow \) complex freq. \( \rightarrow \) IFFT or IDFT \( \rightarrow \) time series
   contains many frequencies encoding information

3. So what is the lowest & highest freq. we can encode?
   Lowest = \( \frac{1}{T} \) since \( T \) is defined in samples.
   Highest = \( \frac{1}{2} f_s \) Nyquist.

3. From the perspective of IDFT.
   \( X(m) \) \( \rightarrow \) IDFT \( \rightarrow \) \( x(n) \)
   freq. samples \( \rightarrow \) N point IDFT \( \rightarrow \) time samples

\[ x(n) = \frac{1}{N} \sum_{m=0}^{N-1} X(m) e^{j2\pi n m/N} \]

(\( N \) samples) \( \rightarrow \) complex freq.
(\( N \) samples) \( \rightarrow \) complex time series
Let's look at different $X(m)$.

- $X(0)$ is DC.
- Setting $X(0)$ to a value shifts the whole time series.

i.e.

$$X_n = \frac{1}{N} \sum_{m=0}^{N-1} X(m) e^{j \frac{2\pi}{N} m n}$$

for $m=0$, $X(m=0)$ will get added like an offset to all values of $X_n$.

i.e.

$$X(n) = X(m=0) + X(m=1). e^{j \frac{2\pi}{N} n} + X(m=2). e^{j \frac{2\pi}{N} 2n} + \ldots$$

This is a constant bias.

Now look at $X(m=1)$.

- This is lowest non-trivial freq.
- If all other $m$'s are 0's, then $X(n)$ will be $X(m=1). e^{j \frac{2\pi}{N} n}$ which is

$$X(n) = X(m=1). (\cos \theta + j \sin \theta). \quad \text{where } \theta = \frac{2\pi}{N} n.$$

- $X(\frac{N}{2})$ we get max freq. sine

$$X(n) = \sum X(m) e^{j \frac{2\pi}{N} \cdot (m=\frac{N}{2}) n}$$

$$X(n) = \sum X_m. e^{j \frac{\pi}{N}}$$

$n\pi$ means the signal switches back and forth.
\( F = \frac{K}{N} f_s \)

\( \downarrow \quad \text{Highest freq} = \text{Nyquist} \)

\( \downarrow \quad \text{mirror freq. or negative freq.} \)

Positive freq. 

Negative freq.

**OFDM Symbol Generation**

\[ X_m = \text{DATA} \quad X_{m=0}, X_{m=1}, \ldots, X_{m=N-1} \]

\[ X(t=0), X(t=1), \ldots, X(t=N-1) \]

\( T \) samples.

Now \( X_m = (a+jb) = \frac{\text{Im}}{\text{Re}} \)

So if Data is 0010 0110 0100 0011

\( X_{m=0} = 1+j1 \)
\( X_{m=1} = 0+j1 \)
\( X_{m=2} = -1+j1 \)
\( X_{m=N-1} = -1-j1 \)
3. How many \( X_m \) should I choose? i.e., what is \( N? \)
   - \( N \) should be a power of 2 so we can do Fast DFT (FFT)
   - Function of the bandwidth \((\text{kHz})\)

4. Bandpass Modulation

   \[ \text{IFFT} \rightarrow X(n) \rightarrow \text{Quadrature Modulator} \]
   - \( X(n) \) complex series at base band
   - \( f_c \) (carrier freq.)

   Transmit
   - Bandpass modulated signal

But note: Complex valued \( X_n \) needs to be transmitted as real valued signals from the antenna.

Quad Mod: Use \( f_m'' \) as the lowest possible freq. (i.e., the modulating signal is greater than \( 2 f_s \))

Idea: Take \( X(n) = \{ x_1, x_2, x_3, \ldots \} \) and interleave the \( \text{Re} \) and \( \text{Im} \) values. i.e., \( \text{Re}\{x_1, x_3, \ldots\}, \text{Im}\{x_2, x_4, \ldots\} \)

So, \( f_m'' \), also denoted as \( f_{\text{mod}} = 2 f_s \)

where \( f_s \) is highest freq. of \( X(n) \). Use \( f_{\text{mod}} \) to modulate \( f_c \).

Data \( X_m \) to IFFT at \( f_s \) to Quadrature Mod. to \( 2 f_s \)

\( f_c = 0.25 \) of normalized freq.
When $\cos + \sin$ combined and sampled at $f_{smod}$, we get

$$\{\text{Re}x[n], \text{Im}x[n], -\text{Re}x[n], -\text{Im}x[n]\} \Rightarrow \text{Real Signal sent over antenna.}$$

However: **FFT** needs to know the start of the symbol accurately. That is:

**In BPSK, its easy**

**Symbol Start Detection Strategies**

1. Cyclic Prefix (or guard interval) - copy last part to the beginning & then do cross correlation with delayed sig.

2. Pilot tones: Set some $X(m)$'s to known values. (e.g., $X(22) = 2$) Then at Rx, we know what to expect for $X_{22}$.
How do you know where symbols start?

Idea: Cyclic Prefix

At Receiver:

Start Detection

Cross correlation to obtain start of symbol.

1. Fails when signal is constant because data is constant.

Cross correlation: \( x_n \) (window of cyclic prefix)

But if received symbol is constant, the cross-correlation also constant \( \rightarrow \) no spikes \( \rightarrow \) failed to detect start.

Ideal signal should be close to random sig so that cross correlation stays at ZERO except when prefix identical.

Example of worst case: \( \text{Data} = [1, 1, 1, \ldots, 3] \) \( \implies \text{IDFT} = [1, 0, 0, 0, 0 \ldots 3] \) delta pulse

Example of best case: \( \text{Data} = [1, 0, 0, 1, 0, 1, 1, \ldots] \) Random sequence

But how to make data sequence look random??
Idea: At Tx

- Known Random Sequence

- XOR

- Output

How to create known random seq.?

→ Simple Idea: Shift Register

- Load a random sequence in a shift register,
- then shift and feedback some function of output bits.

Feedback shift register

- Creates pseudo random seq.
- Repeats after very long seq. (i.e., period very long).
- Output random.

Energy Dispersal spreads energy across the spectrum, which is a good property.

Energy Dispersal

- Data
- XOR
- Randomized Data
- Called "Energy Dispersal"
- IFFT

Data 1,0,1,1,0...

→ XOR

→ IFFT

→ B.P. Q Mod

→ Cyclic Prefix

Cyclic Prefix still not perfect

→ Because thermal noise can make the spike "smudged".

- Cross corr.
- MM Many local maxims.

However, we need this peak to be exactly the correct sample.

Solution

→ Coarse grained start detection from cyclic prefix
→ Use pilot tones to fine tune to one sample
Pilot tones:

Each freq. contains QAM coded data.
Except some contains known amp. & phase called pilot.

Receiver Side: Precise start detection possible because:

- a pilot tone is a complex cos/sin wave,

If you start at wrong sample, the phase will be badly messed up.

\[ \begin{align*}
\text{radius} & \quad M \\
\text{phase} & \quad \cos \\
\text{sin} & \quad \sin
\end{align*} \]

i.e.,

You start wrong with screw up phase.

So strategy:

1. Make pilot tone \( X(k) = 1 + j0 \) \( \Rightarrow \) Real.

2. Then, shift the start of the FFT until the pilot tones are all real.

3. Keep shifting until this is true.