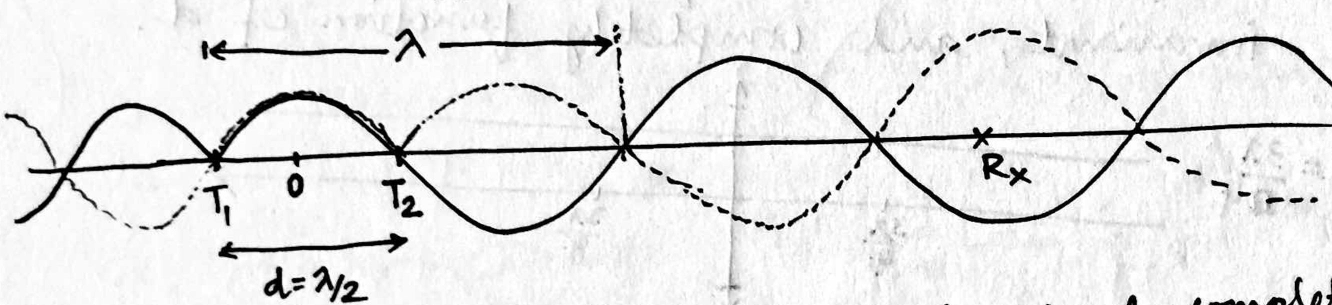
Foundations of Beamforming

- Consider 2 transmitters T_1 and T_2
  - Both transmitting a sinusoid signal of wavelength λ.
  - Also, T_1 and T_2 are separated by distance d.

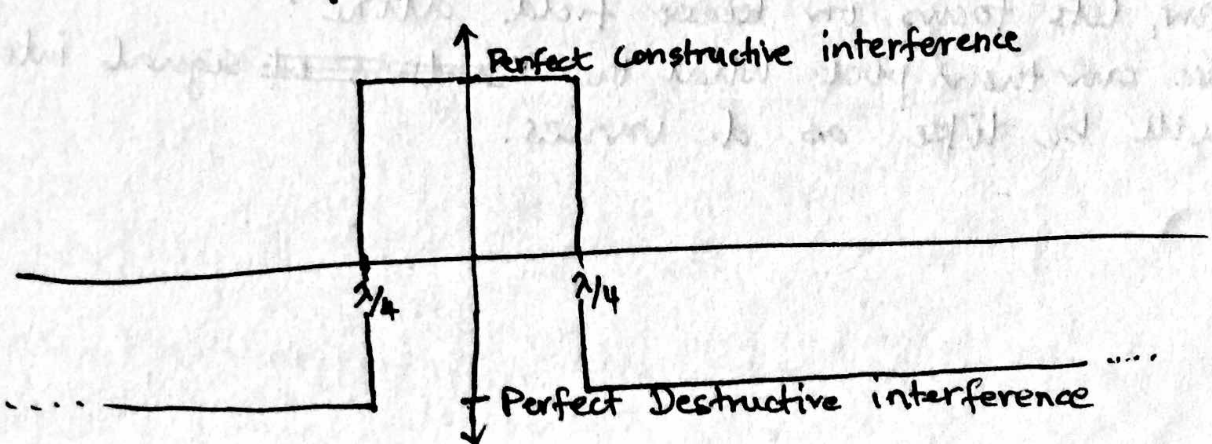
The question of interest is:

How do the two signals interfere with each other (i.e., constructively or destructively) at different locations in a 2D space?

- Simplest case:
  - ↳ 1D space
  - ↳ $d = \lambda/2$



- Clearly, at any location of R_x, the two signals completely get cancelled, provided the location is on the right of T_2.
- Now, between T_1 and T_2, the signals perfectly superimpose, causing constructive interference.
- On the left of T_1, again there is perfect destructive interference.
- This can be graphed as follows:



→ Varying d in 1D case :

- Observe that, ~~regardless of the value of d,~~ for any value of d, the difference in the signal distance Δs is d.

$$\text{Signal distance } S_{T_1} = \text{dist}(T_1, R_x)$$

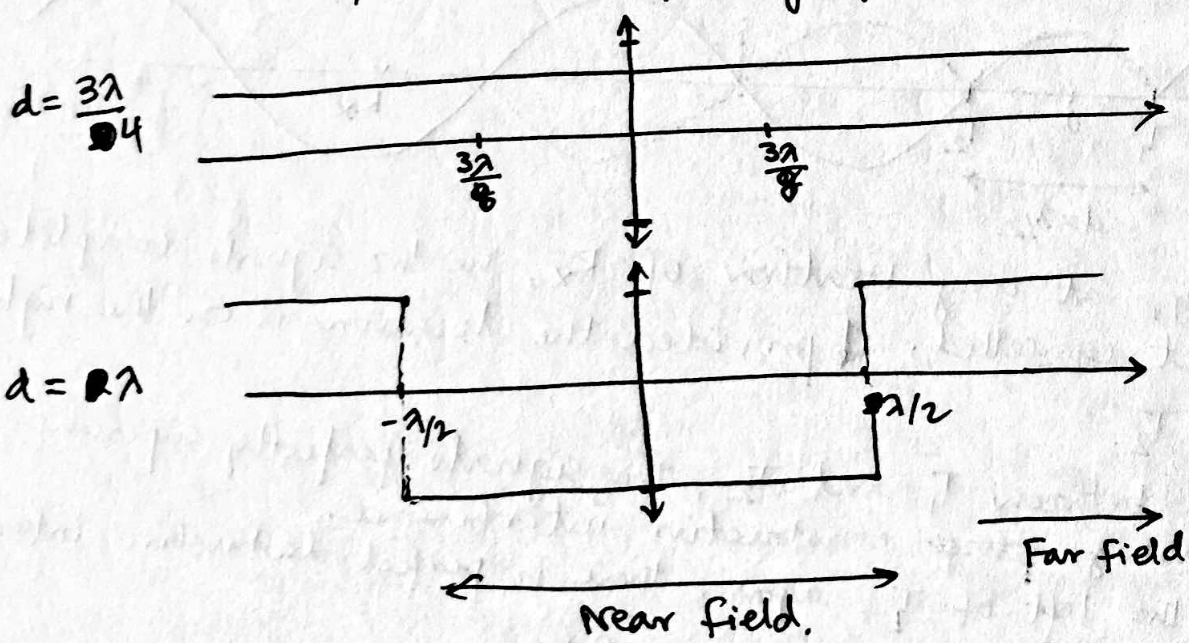
$$\text{Signal distance } S_{T_2} = \text{dist}(T_2, R_x)$$

$$\text{difference } \Delta S = S_{T_1} - S_{T_2}$$

- In other words, as the R_x moves away to ∞, Δs remains the same.

$\therefore$ Phase diff $= \dfrac{\Delta s}{\lambda} \cdot 2\pi$ remains the same.

- Thus, the cancellation quality along 1D line is invariant, and completely function of d.



→ Now, let's focus on ~~near~~ far field alone :
We can then plot what the ~~nature of~~ signal interaction will be like as d varies.

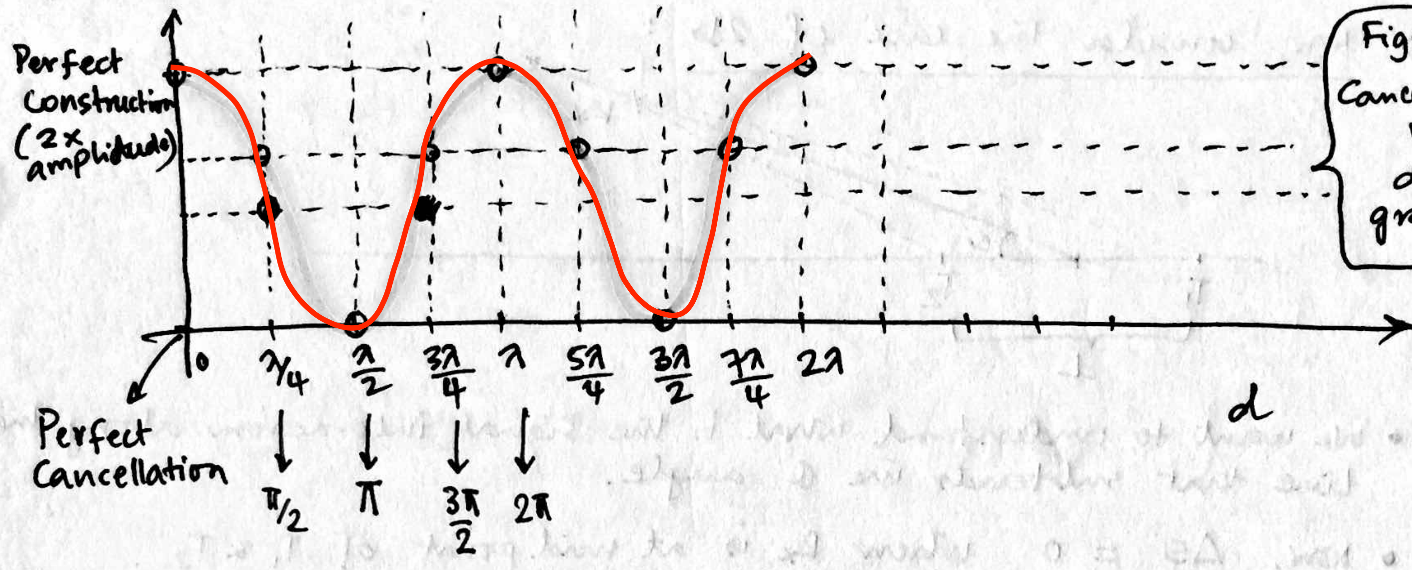
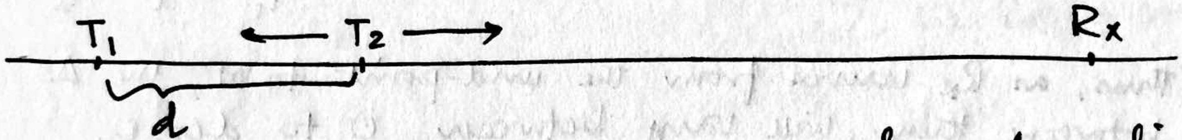


Figure: Cancellation vs. d graph

→ Put differently:

Say we fix T_1 and R_x ~~as follows~~ and move T_2

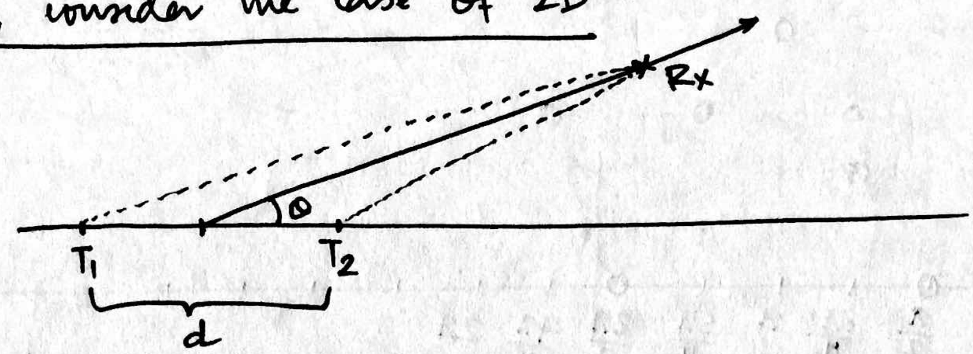


- R_x will experience the orange curve above depending on the value of d between T_1 and T_2.

OR, ~~moving T_2 is equivalent to changing phase at R_x.~~

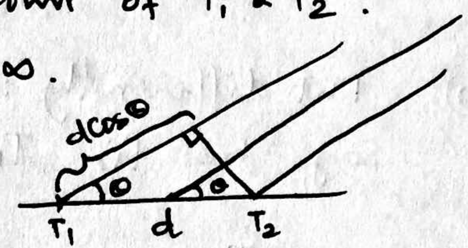
- Any desired phase difference can be created at R_x just by moving T_2.

→ Now, consider the case of 2D :



- We want to understand what is the signal interaction along the line that subtends the θ angle.
- Now, $\Delta s = 0$ when Rx is at mid point of T_1 & T_2.

  $\Delta s = d\cos\theta$ when Rx is at ∞.

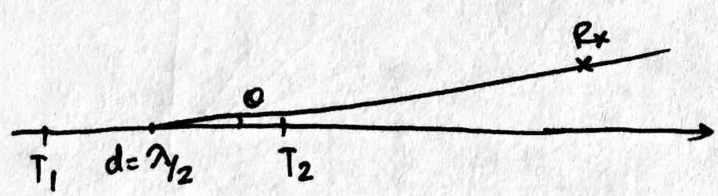


- Thus, as Rx moves from the mid-point to ∞, the Δs between them will vary between 0 to $d\cos\theta$.
- From previous 1D case, we see that $\Delta s = d$ for far field. not near field

  ∴ To understand Rx's experience as it moves along the line, its sufficient to understand what happens when T_2 moves from [$d\cos\theta$ to 0] in the 1D case.
- Since we want to stay in far field, we don't want to move T_2 from $d\cos\theta$ all the way to 0.

  Instead let's move T_2 to some small value close to zero

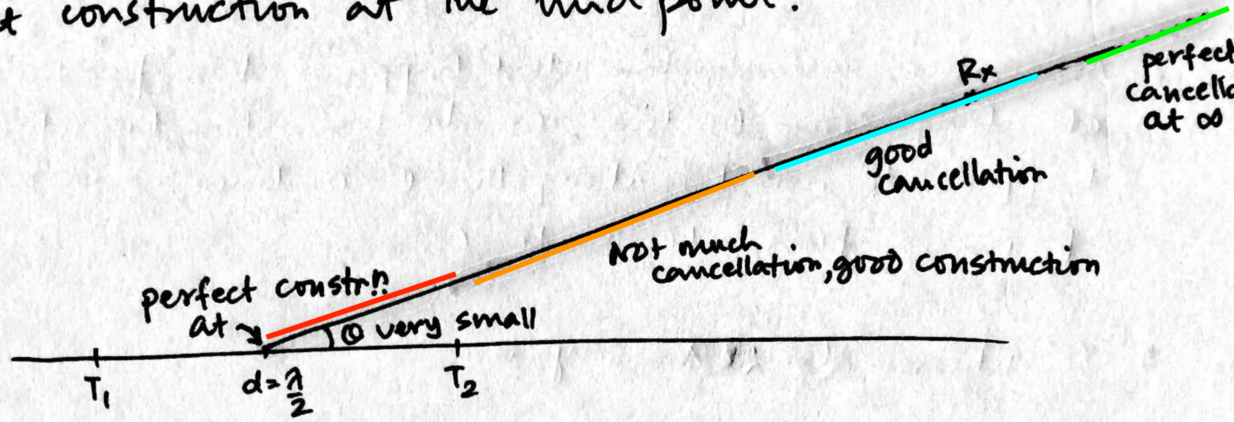
→ Let's start with small θ, and $d = \lambda/2$



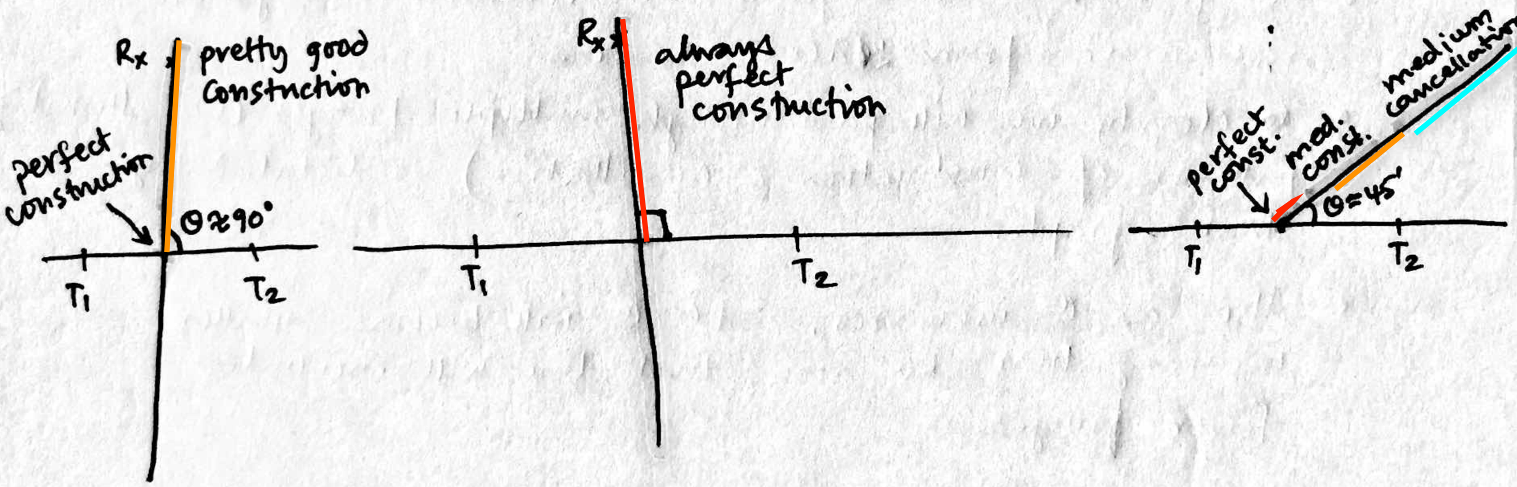
$$d\cos\theta = \frac{\lambda}{2}\cos\theta \approx \frac{\lambda}{2} \text{ for small } \theta.$$

So far away, Rx experiences perfect cancellation, and as Rx comes closer to mid point, it experiences overlaps of two signals, and ultimately achieves perfect construction at the mid point.

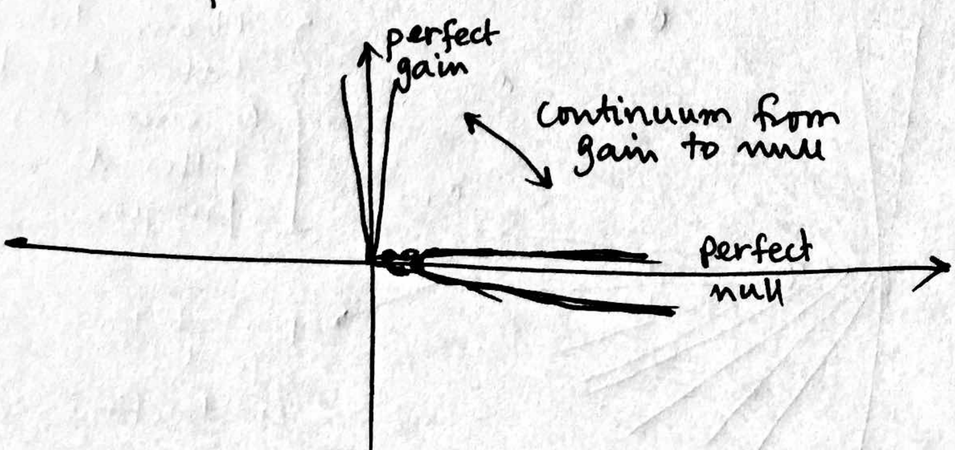
- So



→ As θ increases, $d\cos\theta$ becomes smaller, so as in the graph shown earlier, the signal interaction now becomes closer to perfect construction.
Thus around the bisection of $\langle T_1, T_2 \rangle$, the Rx experiences moderate to good signal construction.



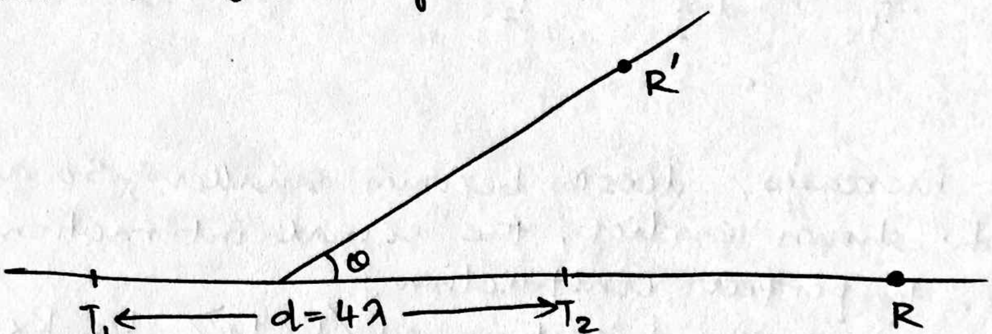
→ So, the "beam pattern" then looks like.



→ Now, consider the case where d varies.

- As earlier, understanding what happens at a ~~far field~~ R_x along some θ line, can be understood by what happens to an R_x along the $\theta = 0$ line, and as d varies from $d\cos\theta$ to 0.

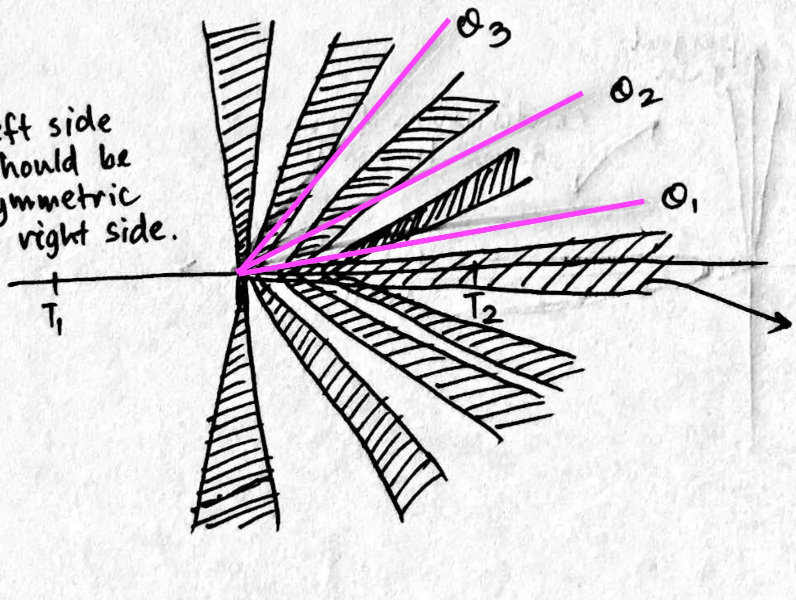
- So when $d = 2\lambda$ say:



- We go back to the original "cancellation vs. d graph" and move from $2\lambda\cos\theta$ to 0.
- Clearly we will pass through multiple fringes (i.e., alternating patterns of construction & cancellation) as dictated by that graph.
- Also, as θ increases, $2\lambda\cos\theta$ will become smaller, meaning that R_x along that line will encounter fewer fringes.

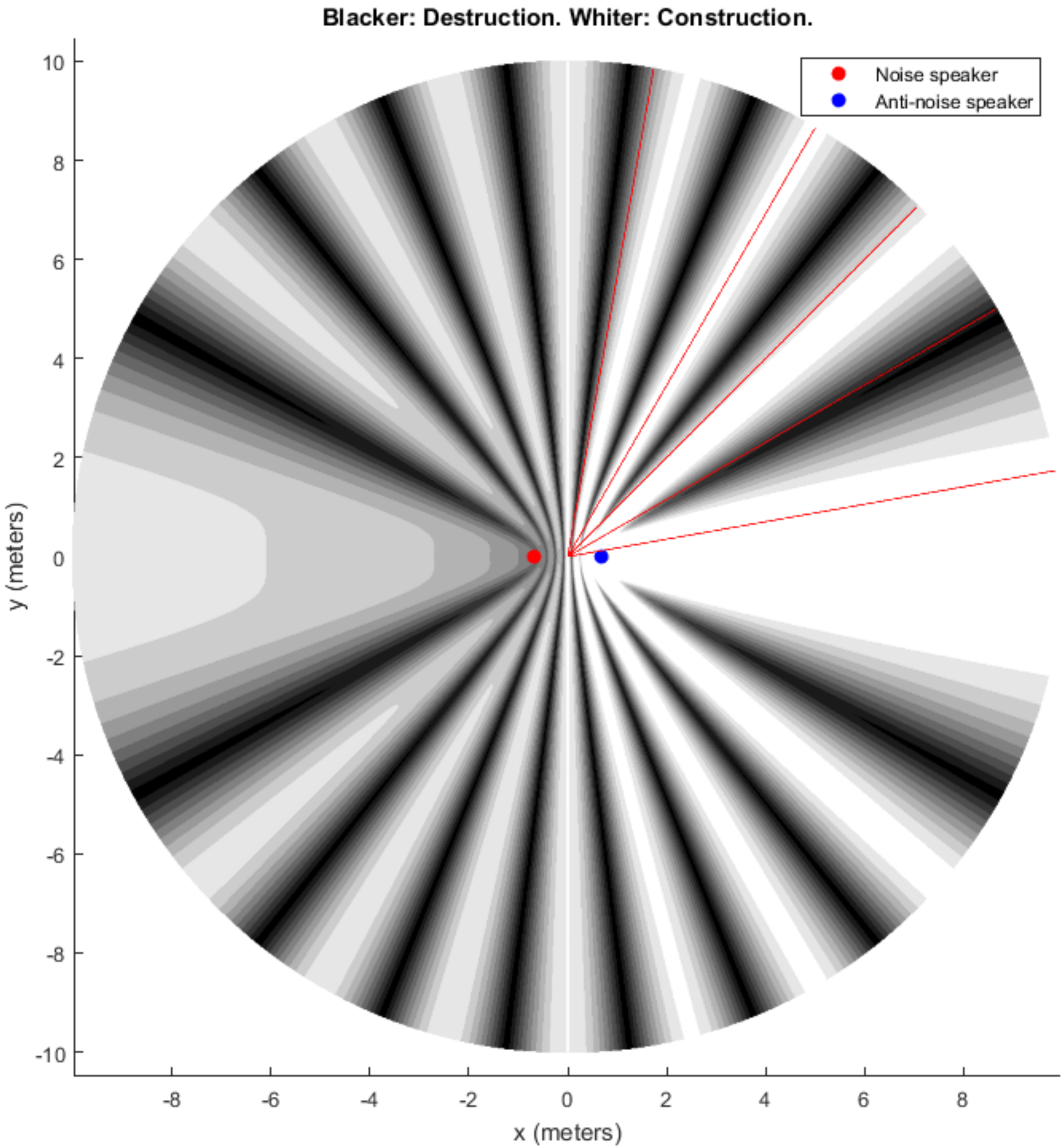
→ The overall radiation pattern at 4λ

Left side should be symmetric to right side.



Observe how different values of θ encountering different no. of fringes.

perfect to good construction


Blacker: Destruction. Whiter: Construction.
Noise speaker
Anti-noise speaker
y (meters)
x (meters)
10
8
6
4
2
0
-2
-4
-6
-8
-10