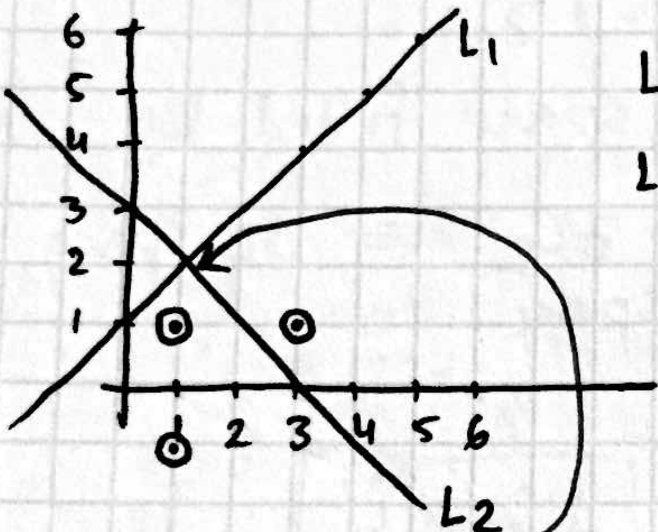
→ Simultaneous Eq^{n}:

$$\left.\begin{aligned} a_1 x + b_1 y &= c_1 \\ a_2 x + b_2 y &= c_2 \end{aligned}\right\} \quad \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Matrix form.

$$A\hat{x} = \hat{b}$$

→ Geometic view



$$L_1 \equiv y = 1.x + 1$$

$$L_2 \equiv y = -1.x + 3$$

$$x + y = 3$$

$$-x + y = 1$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Combine $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ with weights to form $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$\text{weights} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

→ Col. and Row view:

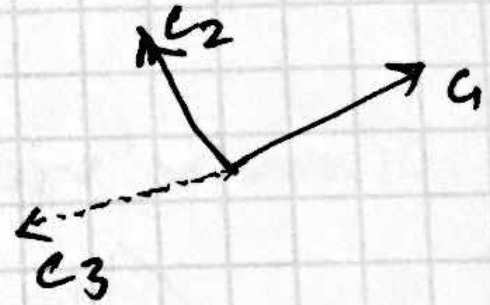
$$\begin{bmatrix} c_1 \end{bmatrix} x + \begin{bmatrix} c_2 \end{bmatrix} y = \begin{bmatrix} | & | \\ c_1 & c_2 \\ | & | \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

or

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} — r_1 — \\ — r_2 — \end{bmatrix} = x\begin{bmatrix} — r_1 — \end{bmatrix} + y\begin{bmatrix} — r_2 — \end{bmatrix}$$

③ Col. & Row space

$$A = \begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix}$$



Col space = Entire space filled by

$$x c_1 + y c_2 + z c_2 \equiv \text{3D space}$$

Similarly row space.

③ Popular form : $\boxed{A \hat{x} = \hat{b}}$

Col. of A

combined by vectors of x

to produce $\vec{b}$

→ Solve $Ax = b$

Gaussian Elimination is convert A to U (upper triang. matrix).

→ Subtracting rows can be expressed as permutation matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{do nothing}$$

$$E_{2-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{subtract row1 from row2}$$

$\therefore \; E_{2-1} \cdot A \equiv$ one variable eliminated

$E_{3-1} \cdot E_{2-1} \cdot A \equiv$ two variables eliminated

$\ominus$ $Ax = b \quad \therefore \quad x = A^{-1} b$

inverse.

means

Permutation performed earlier just needs to be reversed in sign

E_{2-1}^{-1} = just change sign of element $\langle 2, 1 \rangle$

$\ominus$ $A \cdot A^{-1} = I = \cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

How to find A^{-1}

$$\begin{bmatrix} 1 & 5 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 5 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ AND } \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

OR

$$\left[\begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & A^{-1} \end{array}\right]$$

Gaussian Elim.

$\boxed{\begin{array}{l} Ax = b \\ \text{Q: When is this solvable?} \end{array}}$

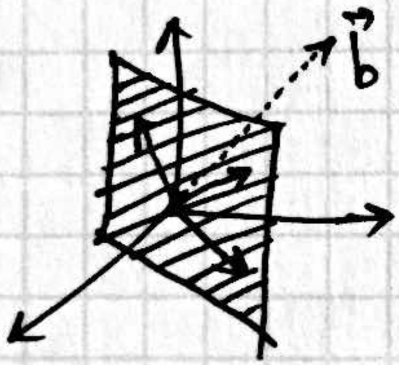
A. When A is invertible.

or A is non-singular,

i.e., $A\hat{x} = 0$ for $\hat{x} \neq 0$

$\Downarrow$

$\vec{b}$ needs to be in the column space of A.



If 3 3D vectors make up 2D space, then any $\vec{b}$ may not fall on that plane.

2 3D vectors can be combined to form 3rd 3D vector.

∴ ~~All ways of combining called null space.~~

∴ Some space remains uncovered called A is singular.

This means columns are not indep.

$\therefore \det(A) = 0$

7 Some basic properties

1. $AB \neq BA$ non-Commutative
2. $A(BC) = (AB)C$ Associative
3. $(AB)^{-1} = B^{-1}A^{-1}$
4. $(AB)^T = B^T A^T$
5. Symmetric matrix when $A^T = A$
6. $A^T A$ always symmetric